Experimentally Measuring the Earth's Mass

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Abstract

The current understanding of gravity comes from Newton's universal law of gravitation. As he stated it, the force of gravity is proportional to the product of two masses and inversely proportional to their distance squared. This statement allows for an understanding of the theoretical concept of gravity, but does not allow for calculations of the force between the two bodies exactly. It is not know, until now, to a precise measurement, what the factor G by which these quantities are proportional. G was calculated as G was calculated in a laboratory setting, using masses suspended from a torsion pendulum attracted to outside masses, as 5.61×10^{-11} kg m s^{-1}. Using this constant the Earth's mass can be calculated as 7.09×10^{24} kg.

1 Introduction

In 1687 Sir Issac Newton published the Philosophiæ Naturalis Principia Mathematica [1]. Within this publication Newton goes on to describe the law of universal gravity by stating that any two bodies are attracted each other with a force proportional to the product of the two masses, and inversely proportional to the square of their distance from each other. As of 1798, the constant associated with this law was not known precisely. In the literature of the time, there was no mention of the gravitational constant G. Rather, solutions for the density of the Earth were considered instead. In 1692 Edmond Halley postulated that the earth was hollow as a solution to anomalous compass readings[2]. This was later disproven in 1749 by Pierre Bouguer[3]. Bouguer conducted an experiment to find an initial estimate of the earth's density by finding the length of a pendulum with fixed period at several altitudes. A more accurate prediction was made in 1778 by Charles Hutton by observing the deflection of a pendulum on either side of a mountain [4]. Through measuring this deflection and estimating the mass of the mountain based off of its volume and the density of it's surface rocks, an estimate for the earth's density was made. This study presents a method for precisely calculating the universal gravitational constant G, and by extent the Earth's mass, in a lab environment.

2 Methods

The gravitational constant G is observed by calculating the the force that two large masses apply to two smaller masses connected to a torsion pendulum. In Figure 1 these are the larger circles outside of the box. When these large masses are added to the apparatus the torsion pendulum starts to oscillate until it reaches equilibrium. At equilibrium the large masses are applying a torque of

$$\tau_{grav} = 2F_{grav}D \qquad (1)$$

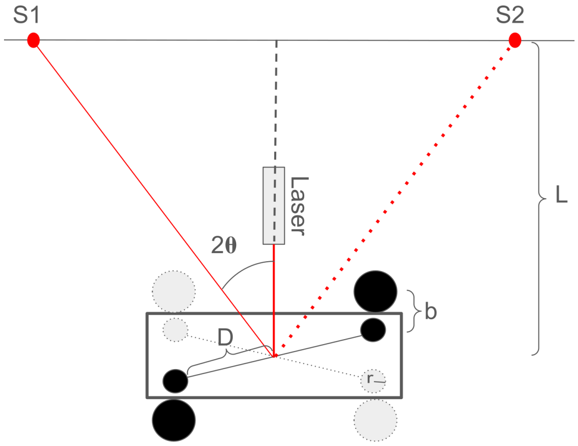

Figure 1: The apparatus for this experiment. The apparatus consists of a two masses attatched to a rod that is suspended on a torsion pendulum. Attatched to the pendulum is also a mirror. The mirror reflects light from a laser beam and allows small changes in the deflection of the pendulum to be recorded. There are also two larger masses that can easily swivel from one position to another. There are two states that the apparatus can exist in. The first position is shown in solid lines and black, and the second position is shown in dotted lines and grey.

where D is half the length of the pendulum arm.

$$F_{grav} = G\frac{m_1 m_2}{b^2} \tag{2}$$

where m_1 and m_2 are the mass of one small mass and one large mass. b represents the distance between the center of mass of each small mass large mass pair. This torque is counteracted by an equal and opposite torque from the torsion pendulum where

$$\tau_{pendulum} = -k\theta \tag{3}$$

Here k is the torsion constant of the apparatus and theta is the angular deflection of the pendulum. Theta can be calculated by determining the total deflection of a laser that is reflected off of a mirror that is attatched to the pendulum and is coplanar with the pendulum's arms. The laser's deflection is measured by measuring the total spatial deflection of the laser's intersection with the wall between position 1 and 2. Using the small angle approximation, the angular deflection of the laser $\Delta\theta_{laser}$ can be calculated as

$$\frac{\Delta S}{2L} \tag{4}$$

where L is the distance between the mirror and the wall. Because of the mirror, the angle we are measuring is twice the angular deflection of the pendulum arms. This gives us the final equation

$$\Delta\theta_{pendulum} = \frac{\Delta S}{4L} \tag{5}$$

k can be calculated as

$$k = 2m_2\left(D^2 + \frac{2}{5}r^2\right)\left(\omega^2 - \frac{1}{\tau^2}\right) \tag{6}$$

where ω and τ can be found by fitting the equation for a damped harmonic oscillator to the motion of the spatial deflection of the laser, r is the radius of the smaller mass and m_2 is the mass of the smaller mass. The final equation for G is

$$G = \frac{\left(D^2 + \frac{2}{5}r^2\right)\left(\omega^2 - \frac{1}{\tau^2}\right)\Delta S b^2}{4Lm_1 D} \tag{7}$$

where m_1 is the mass of one of the larger masses.

3 Results

In Figure 2 is a plot of the spatial deflection of the laser against time. This plot shows that the torsion pendulum fits the model for a damped harmonic oscillator with centered on the eventual equilibrium position. Using ω and τ Equation 7 can be solved. G was calculated to be 5.61×10^{-11} kg m s^{-1}. The experiment was conducted with 2 other pairs of large masses. The calculations for G with these pairs used the same ω and τ found from fitting a damped harmonic oscillator in the first experiment. This resulted in calculations for G of 5.69×10^{-11} kg m s^{-1} and 5.71×10^{-11} kg m s^{-1}. Using this new value for G a calculation for Earth's mass can be found to be 7.09×10^{24} kg.

4 Conclusion

In this experiment the force applied to two small masses suspended on a torsion pendulum from the gravitational force of two larger stationary masses was measured. From this measurement a calculation for the universal gravitational constant G was found to be 5.61×10^{-11} kg m s^{-1}. The calculation for G lead to a calculation of the Earth's mass as 7.09×10^{24} kg. This calculation for G differs from Charles Hutton's 1778 calculation of $\sim 8 \times 10^{-11}$ kg m s^{-1}. This calculation of G allows us to get understanding of the mass and density of our own planet as well as astronomical bodies. Data and calculations can be found at https://github.com/Parkerwise/cavendish-data/tree/main.

References

[1] Issac Newton. Philosophiæ Naturalis Principia Mathematica, 1687.
[2] Edmond Halley. An account of the cause of the change of the variation of the magnetic needle; with an hypothesis of the structure of the internal parts of the earth, 1692.
[3] Pierre Bouguer. La Figure de la Terre, 1749.
[4] Charles Hutton. An account of the calculations made from the survey and measures taken at Schehallien, in order to ascertain the mean density of the Earth, 1778.

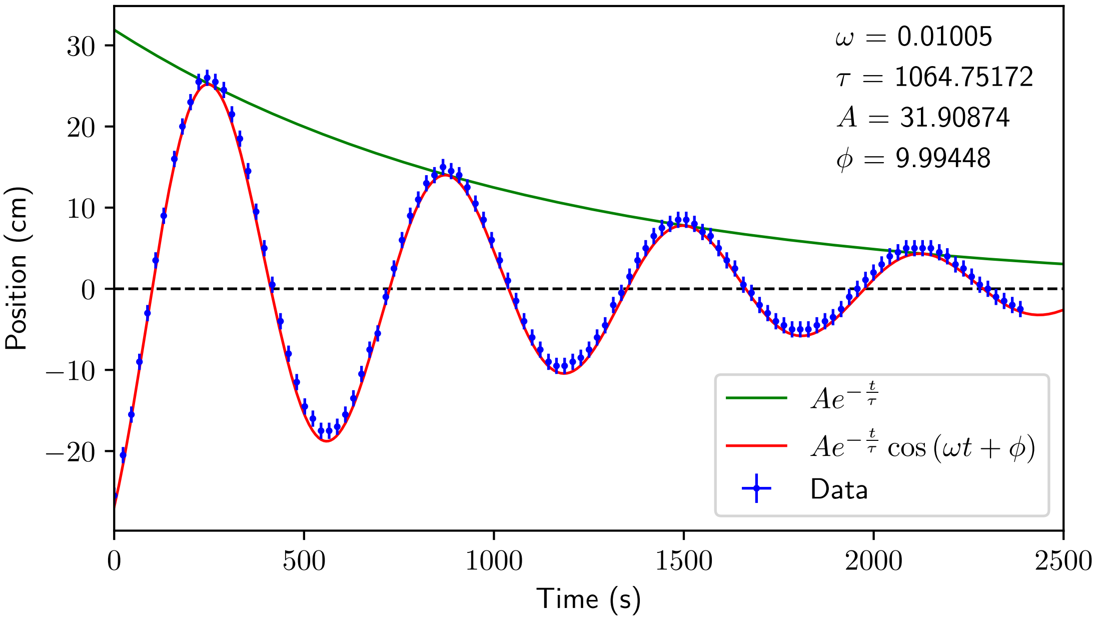

Figure 2: In blue is the plot of the spatial deflection of the laser against time. The positions have been shifted so that $x = 0$ at the equilibrium point. In red is the fit of the data to the model for a damped harmonic oscillator. The model for the exponential decay of the oscillations is graphed in green.